AN ANALYSIS OF SURFACE WAVES GENERATED BY A SUBMERGED HYDROFOIL

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1953

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AN ANALYSIS OF SURFACE WAVES
GENERATED BY A SUBMERGED
HYDROFOIL

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the requirements for the degree of

NAVAL ENGINEER

From the

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(1953)

ABSTRACT

TITLE: An Analysis of Surface Waves Generated by a Submerged Hydrofoil.

AUTHORS: Lieutenant Cowan E. Jones, Jr., U. S. Navy
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Submitted to the Department of Naval Architecture and Marine Engineering on 25 May 1953 in partial fulfillment of the requirements for the degree of Naval Engineer.

This investigation is a study of the characteristics of the surface wave generated by a submerged hydrofoil.

The experimentation is conducted on essentially a two-dimensional basis. Measurements were taken along the centerline of a circulating water channel. The wave generator is an infinite aspect-ratio foil of NACA 4412 designation. Generated under controlled conditions of hydrofoil angle of attack, depth of submergence, and stream velocity, the wave is defined by measurements of basic dimensions such as amplitude and wave length.

Results obtained are:

1. $\lambda = \frac{2\pi V^2}{g}$ as predicted by theory.

2. Curves expressing the relationship: amplitude versus angle of attack, submergence and velocity.

It is concluded that deep water waves can be simulated in a circulating water channel. An extension of the range of this type of experimentation can lead to a complete solution to the characteristics of surface waves generated by a submerged hydrofoil.

20521

Cambridge, Massachusetts
25 May, 1953

Secretary of the Faculty,
Massachusetts Institute of Technology,
Cambridge, Massachusetts.

Dear Sir:

In accordance with the requirements for the degree of Naval Engineer, we submit herewith a thesis entitled "An Analysis of Surface Waves Generated by a Submerged Hydrofoil."

Respectfully,

NOTATION

a. Wave amplitude.

C. Chord-length of hydrofoil.

d_0 Depth of the stream in the flume.

d_1 Depth of submersion of the hydrofoil, measured from the surface to the tip of the leading edge.

F. Froude no. of the flume $\frac{V}{\sqrt{gd_0}}$.

F_1. Froude no. of the hydrofoil $\frac{V}{\sqrt{gc}}$.

H. Manometer head (feet)

L. Width of flume.

Q. Flow rate of the flume (cubic feet/second).

V. Velocity of flow (feet/second).

α. Angle of attack of the foil.

λ. Wave-length.

l_1 Horizontal distance from the hydrofoil leading edge to the first wave hollow.

l_2 Horizontal distance from the hydrofoil leading edge to the first wave crest.

y_0 The vertical distance between the undisturbed stream surface and the first wave hollow.

y_1 The vertical distance from the undisturbed stream surface to the peak of the wave formed above the front of the foil (when occurring).

TABLE OF CONTENTS

I. INTRODUCTION

The use of hydrofoils attached to the hull of a surface vessel is not a new idea. Their application has been attempted in many ways. In the latter part of the last century, Alexander Graham Bell designed a small high-speed craft equipped with foils which attained remarkable speeds for the power then installed. During World War I, the British Admiralty investigated the possibility of using foils on ships with the idea of lifting a ship bodily out of the water to reduce its susceptibility to torpedo attack, but investigations were abandoned without conclusive results.

The Denny-Brown Stabilizer, which was first introduced commercially in the 1920s, represents a successful application of hydrofoils on surface vessels. The stabilizer consists of a hydrofoil located on each side of a vessel's hull at the turn of the bilge. The hydrofoils are actuated by machinery within the ship which causes them to rotate to counteract and reduce the roll of the vessel in a seaway.

For the past fifteen years hydrofoils for use on high-speed surface craft have become increasingly popular. However, their use has been restricted to very small high-speed craft whose displacement is small enough to permit the foils to lift the craft out of the water. With the exception of the Denny-Brown Stabilizers, no real attempt has been made to apply hydrofoils to large ships.

The attachment of hydrofoils to a ship's hull for the purpose

of reducing wave resistance is a comparatively new idea. In a recent paper, read before the 1953 meeting of the Society of Naval Architects and Marine Engineers, Professor M. A. Abkowitz of M.I.T. presented his ideas and the results of his experiments showing the possibility of reducing the wave resistance of ships by the use of hydrofoils located at the forefoot. Under Professor Abkowitz's supervision, towing tank model experiments have been conducted which have shown qualitatively a decrease of model resistance at high speeds.

The objective of this use of hydrofoils is to achieve a net decrease in resistance by accepting increased frictional resistance in return for a large reduction of wave making resistance.

Wave resistance can be considered essentially a pressure phenomenon in which the pressure gradient around a body moving near or on the free surface of a fluid results in the formation of a system of gravity waves. The most prominent, in the case of a surface hull of large displacement, is the bow wave. A reduction in the amplitude of the generated waves by means of a hydrofoil attached to the hull represents a lower energy loss from the moving vessel, which may result in an increase in speed or a reduction in the required horsepower for a proposed design. The presence of a hydrofoil in the vicinity of a vessel's bow, so located that its generated wave system would partially cancel the ship's generated waves (particularly the bow wave), could reduce the wave making resistance of the vessel. Furthermore, as a secondary advantage, this device, by its very location, has the useful characteristic of reducing the pitching of a vessel in a seaway.

Once the form of the hydrofoil has been decided upon, the problem of locating it with respect to the ship so as to achieve optimum wave reduction immediately presents itself. It is therefore necessary that the characteristics of a hydrofoil operating near the surface of a fluid be available. The purpose of this investigation is to analyze the behavior of a typical hydrofoil and determine its characteristics. In a literature survey conducted by the authors, it became obvious that information pertaining to hydrofoil characteristics in producing surface waves was not available. Certain work has been done in regard to the analysis of surface waves generated by a few geometrical shapes (principally bodies of revolution) and the general characteristics of surface waves have been theoretically defined. However, no work has been done on hydrofoils. Therefore, it was decided to examine the characteristics of a wave system generated by a particular hydrofoil, operating at various angles of attack, velocities, and depths of submergence.

Attempts to analyze wave generation by hydrofoils previously had been made in the towing tank at M.I.T., but the results indicated that a towing carriage rather than the installed towing-wire would be necessary for extensive study. The best method available appeared to be a two-dimensional analysis in the circulating water channel installed in the Hydrodynamics Laboratory at M.I.T.

II PROCEDURE

The wave profile was obtained by running a centerline traverse the length of the test section. The surface elevation at each point was obtained by the point gage; the horizontal distances were fixed by alignment of the telescope cross hairs on the probe tip and reading of location on the scale affixed to the telescope bench. The location of the probe tip may be measured to 0.01 centimeter with such a point gage, and the telescope location read to .02 inches. The instrumentation, simple as it may seem, is very precise in comparison to the inherent fluctuation in a circulating water channel.

The profile points were taken at intervals consistent with the wave length and amplitude, and the curve fixed by these points was the basic result of each run. Because of the large number of runs necessary, averaging of several readings of surface elevation at each point was not feasible. The averaging was done by eye, and only one reading obtained at each point. To avoid errors in instrument reading, profile points were plotted as they were obtained, and examination of the resulting curves showed that this method gave sufficient precision, (see sample profiles appendix C).

In conducting the runs, velocity (V), angle of attack (α), depth of submergence (d_1) and total depth of flow (d_0) were controllable.

To analyze the resulting wave, three types of flow were considered, 1) approach flow, 2) transition zone, 3) steady state wave formed after transition. Wave length of the steady state portion (λ),

wave amplitude in the steady state region (a), and the characteristic dimensions of the transition region (y_0, y_1, l_1, l_2) comprised the data obtained from evaluation of the profiles.

Three types of run were taken, according to the data desired.

1) Full profile the length of the test section or to the point where instability of flow made measurements doubtful. From such a profile, all variables could be measured.

2) A profile from undisturbed approach flow through the transition zone, and surface elevation only of the steady state flow at maximum and minimum points. This yields all data except wave length and shape of the steady state portion.

3) Measurement of elevation of approach flow and of the maxima and minima in the steady state wave. This gives only amplitude of the steady wave.

Evaluation of Data.

Variables were obtained in the following manner.

Type 1 runs) "λ" and "a" represent an average of all steady state waves obtained in each run. This, plus the averaging implicit in drawing a smooth curve through the plotted points, yielded reasonable, consistent results. For the parameters of the transition region, we have, of course, but one measurement per run. Fortunately, the transition zone, free from wall and side support effects, is highly stable, and profile measurements to the order of accuracy of the approach flow were obtainable.

Type 2 runs) "a" is the difference of the averages of maxima and minima. The transition zone is evaluated as in type 1 runs.

Type 3 runs) "a" only is evaluated and is, as before, the difference of the averages of maxima and minima.

III RESULTS

1. There is no measurable damping present over the length of the test section or any discernable damping to the point where weir overfall occurs. As high as ten wave-lengths were observed with no apparent change of amplitude or wave-length.

2. It is possible to simulate deep water effects in a circulatory water channel. See Figure XIII.

3. There is no rise of the surface above the foil section at submergences greater than .95 C.

4. There is no effect on approach flow more than one chord-length ahead of the foil.

5. The theoretical relation

$$\lambda = \frac{2\pi V^2}{g} \qquad (1)$$

is confirmed. A plot of this is shown in Figure II .

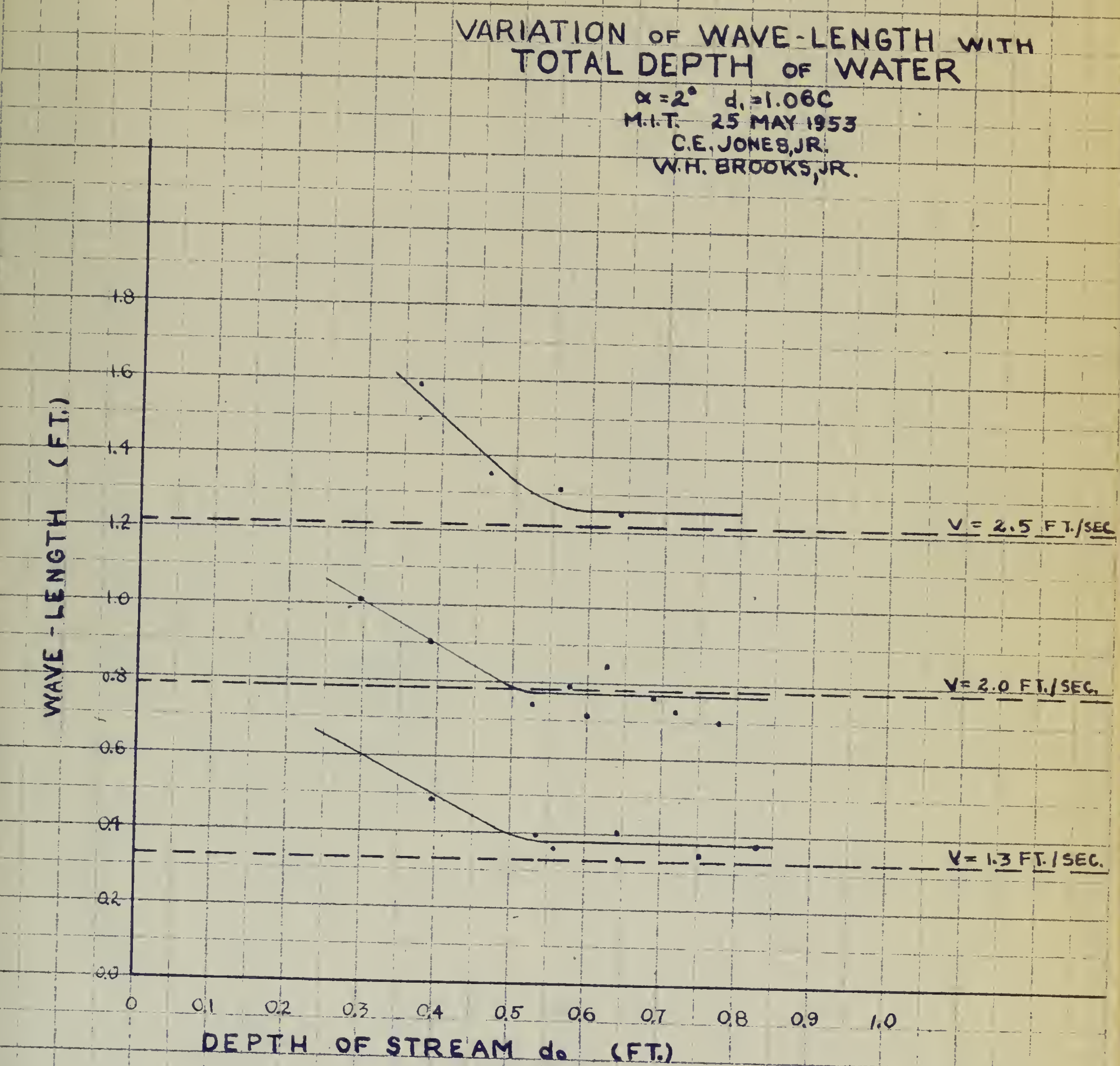
6. Curves of a/c versus F at various angles of attack.

7. Curves of a/c versus d_1/c at various Froude numbers and angles of attack.

8. The following numerical averages and range of variations from these averages were found relating the transition zone to the steady wave:

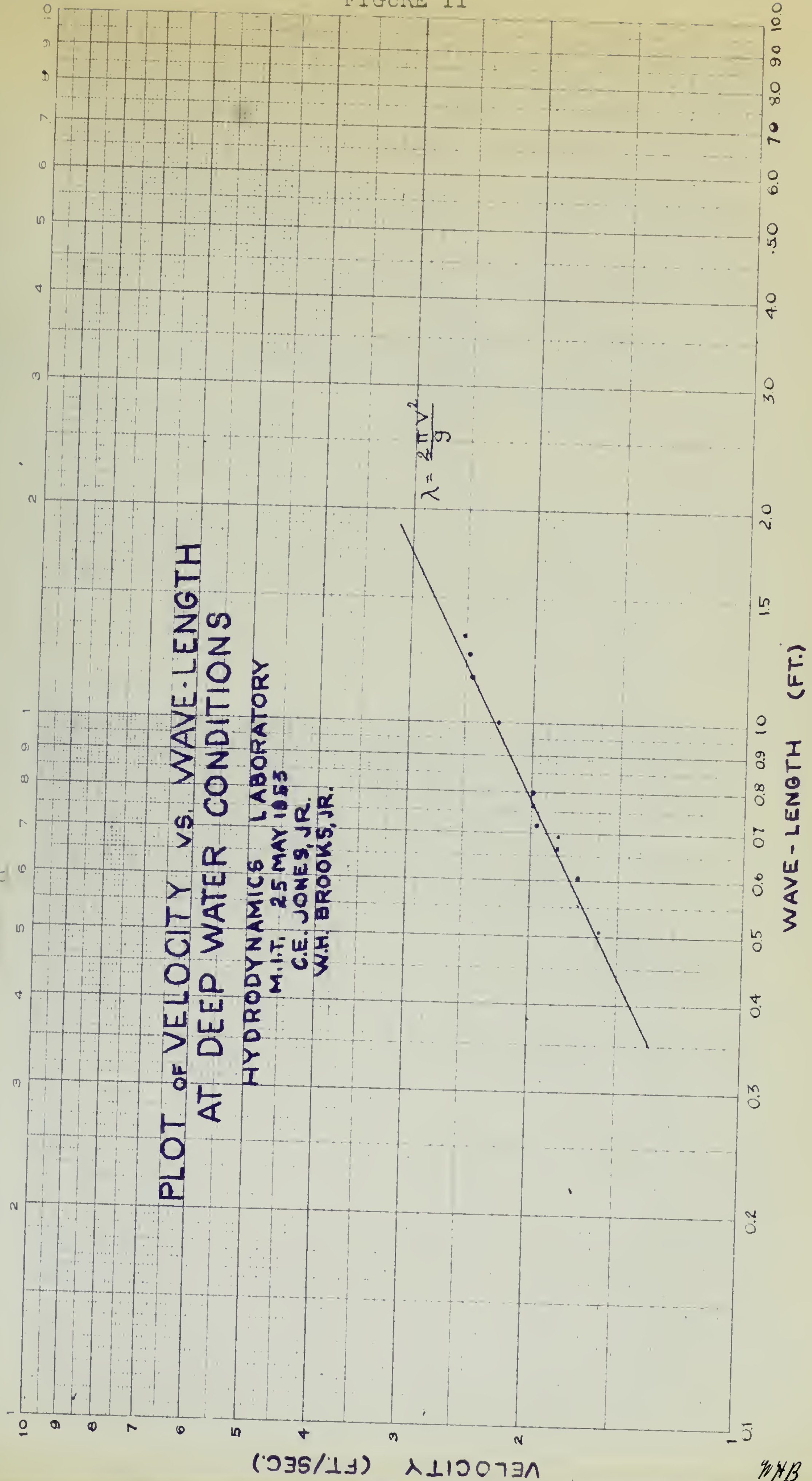
	y_o/a	L_1/λ	L_2/λ
Avg.	.533	.50	1.04
Min.	.44	.46	0.95
Max.	.67	.54	1.14

FIGURE I



WHB CEJ

FIGURE II



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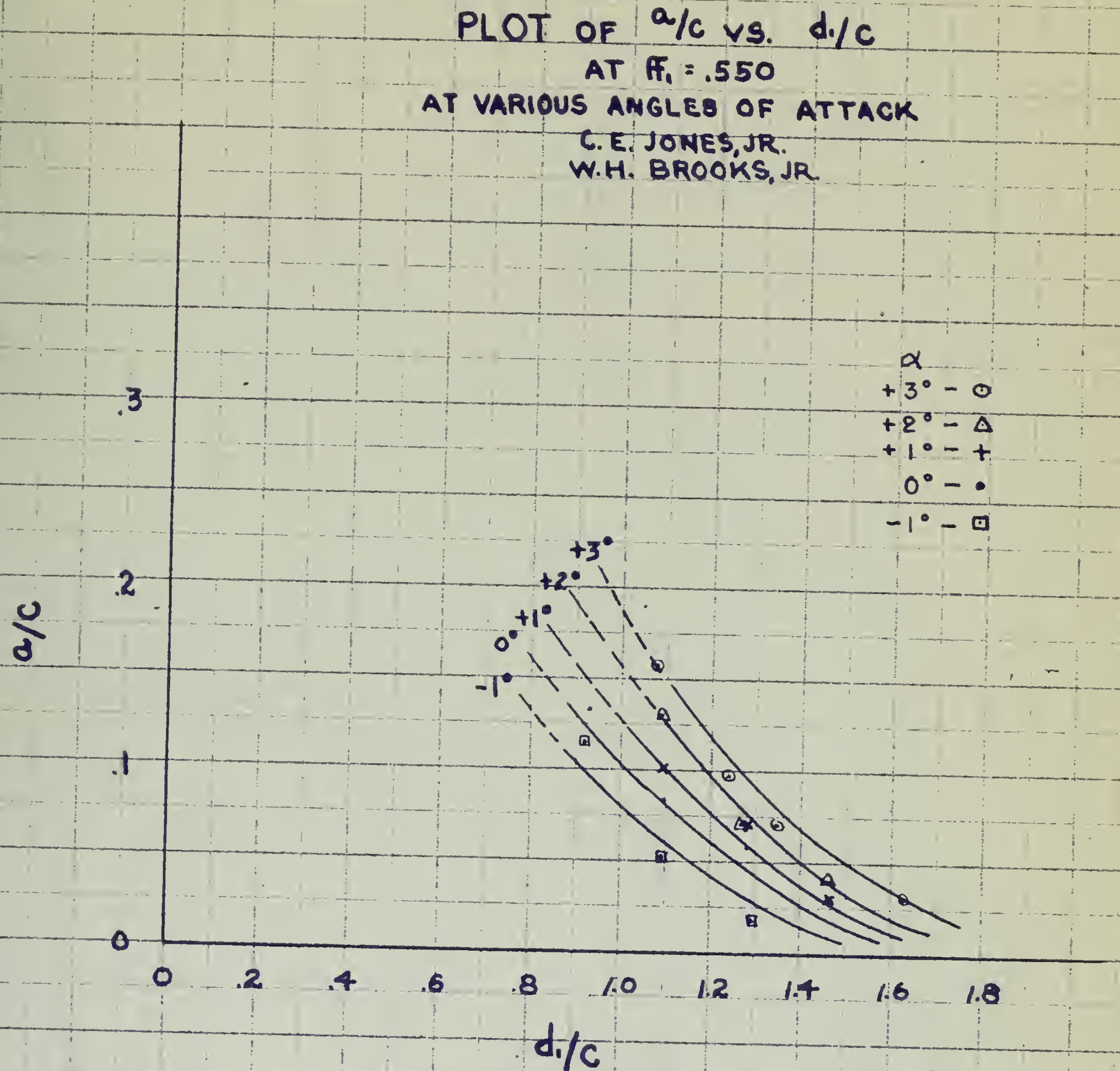

25 MAY 1953
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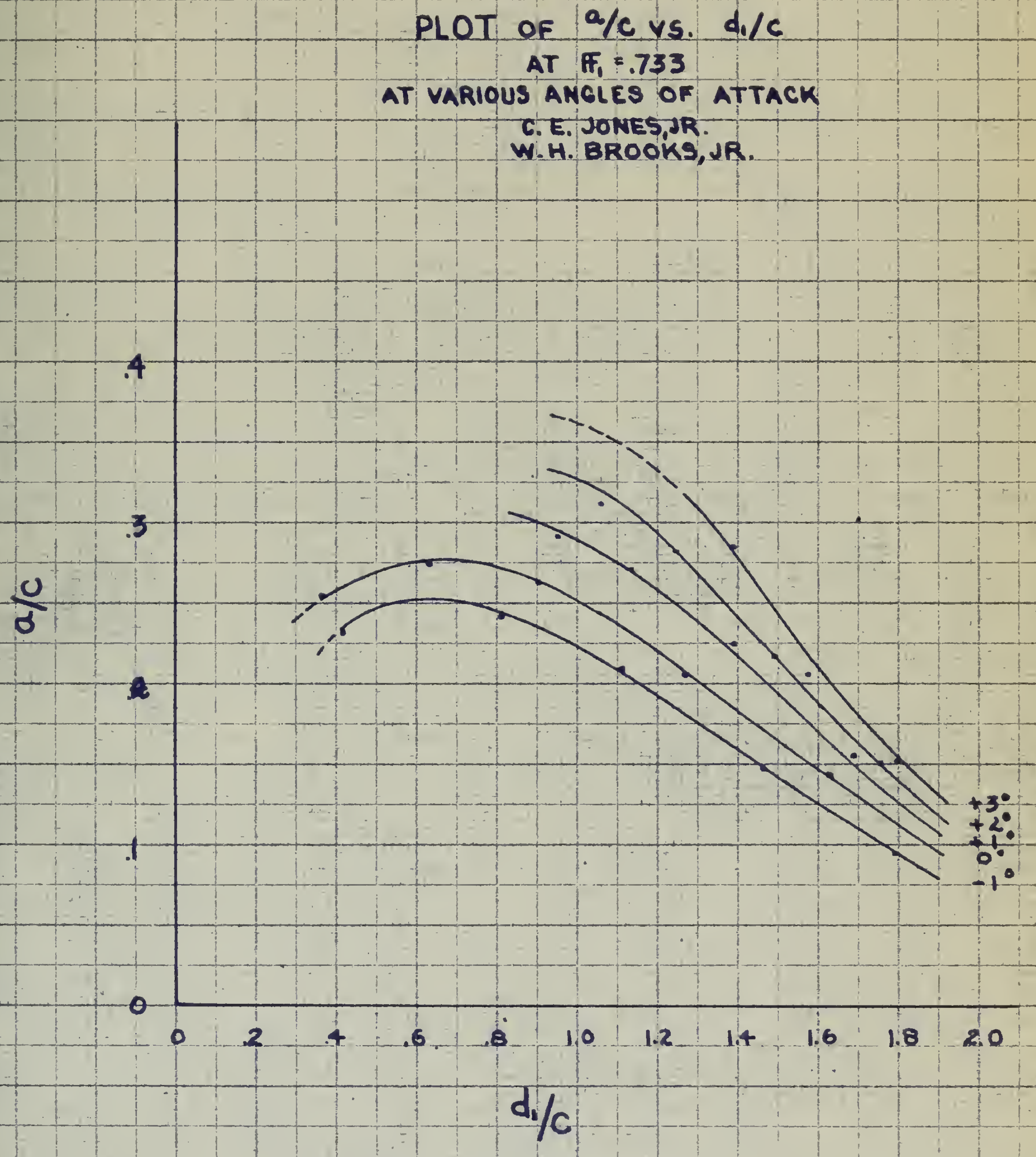

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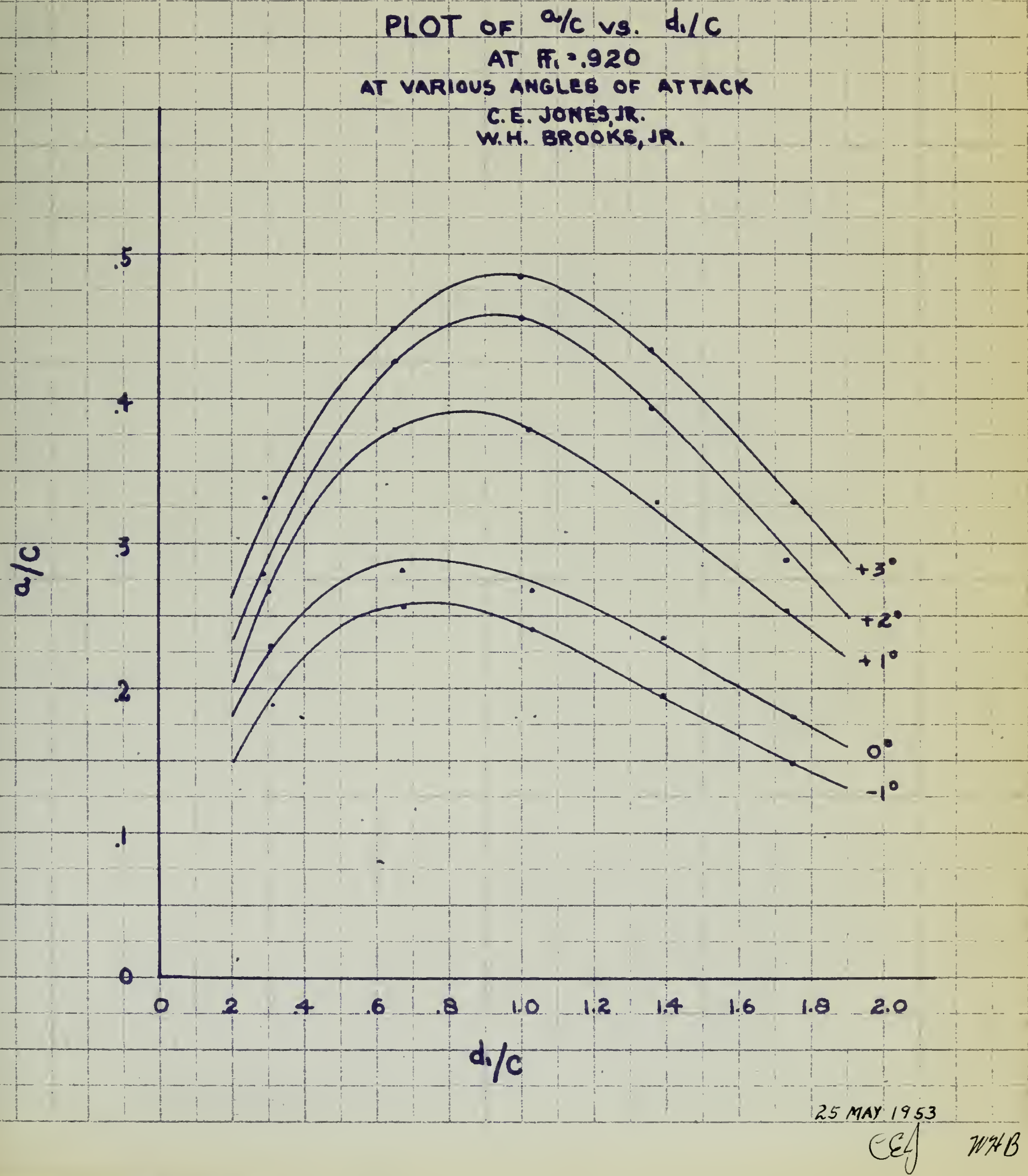
PLOT OF a/c vs. d₁/c
AT F₁ =.920
AT VARIOUS ANGLES OF ATTACK
C.E. JONES, JR.
W.H. BROOKS, JR.
a/c
0
.1
.2
.3
.4
.5
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
d₁/c
+3°
+2°
+1°
0°
-1°
25 MAY 1953
CEJ
WHB

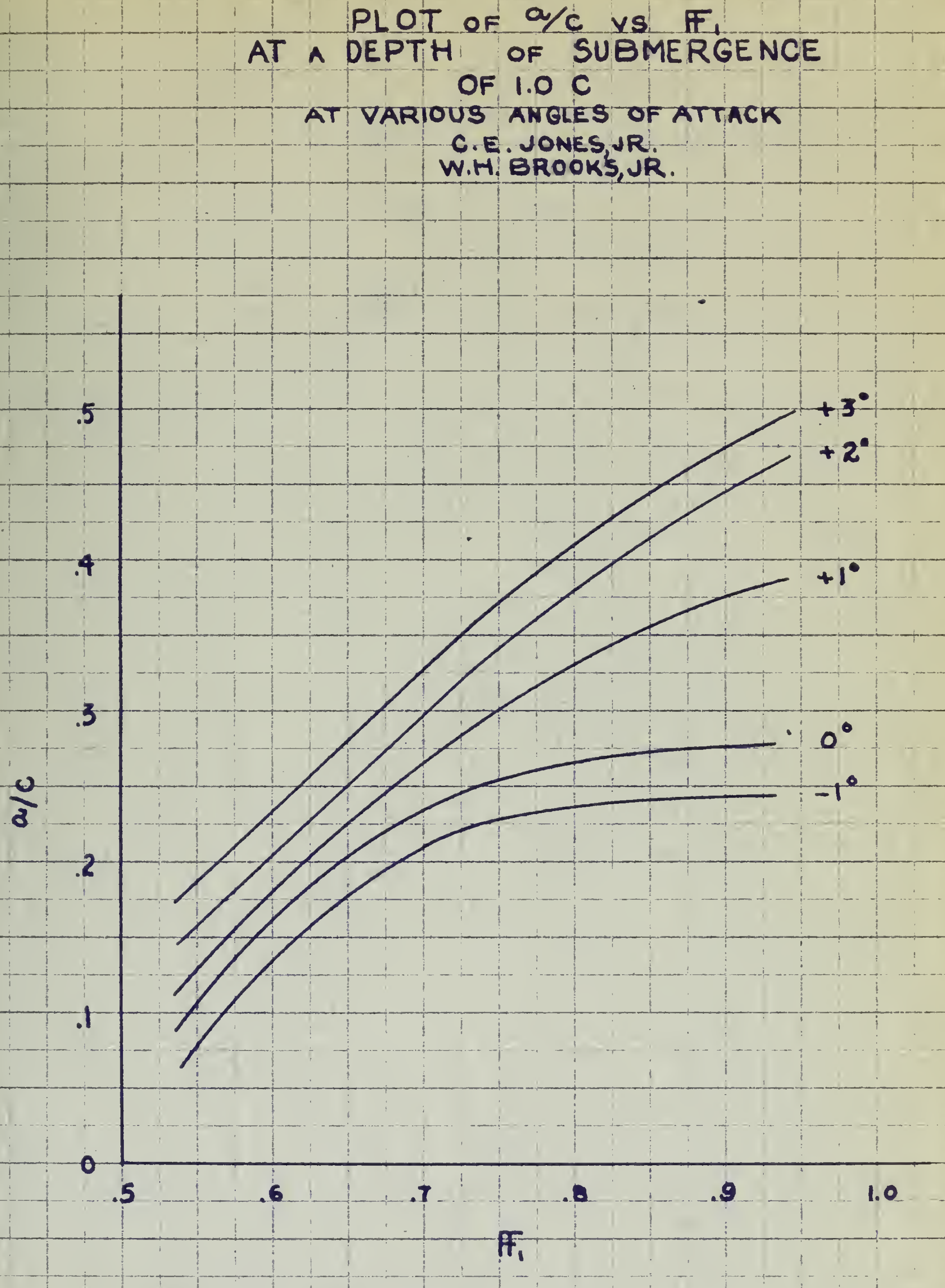

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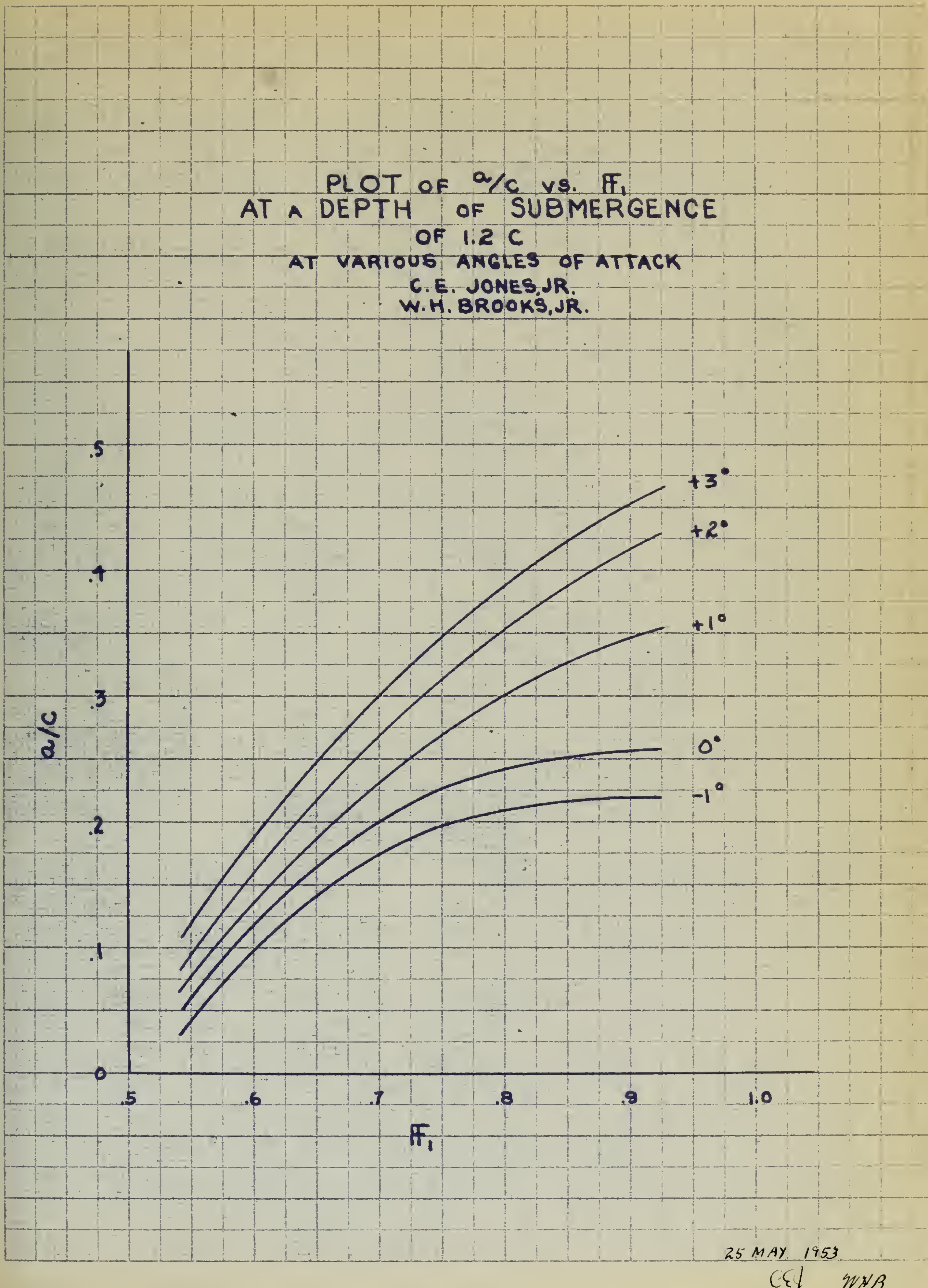
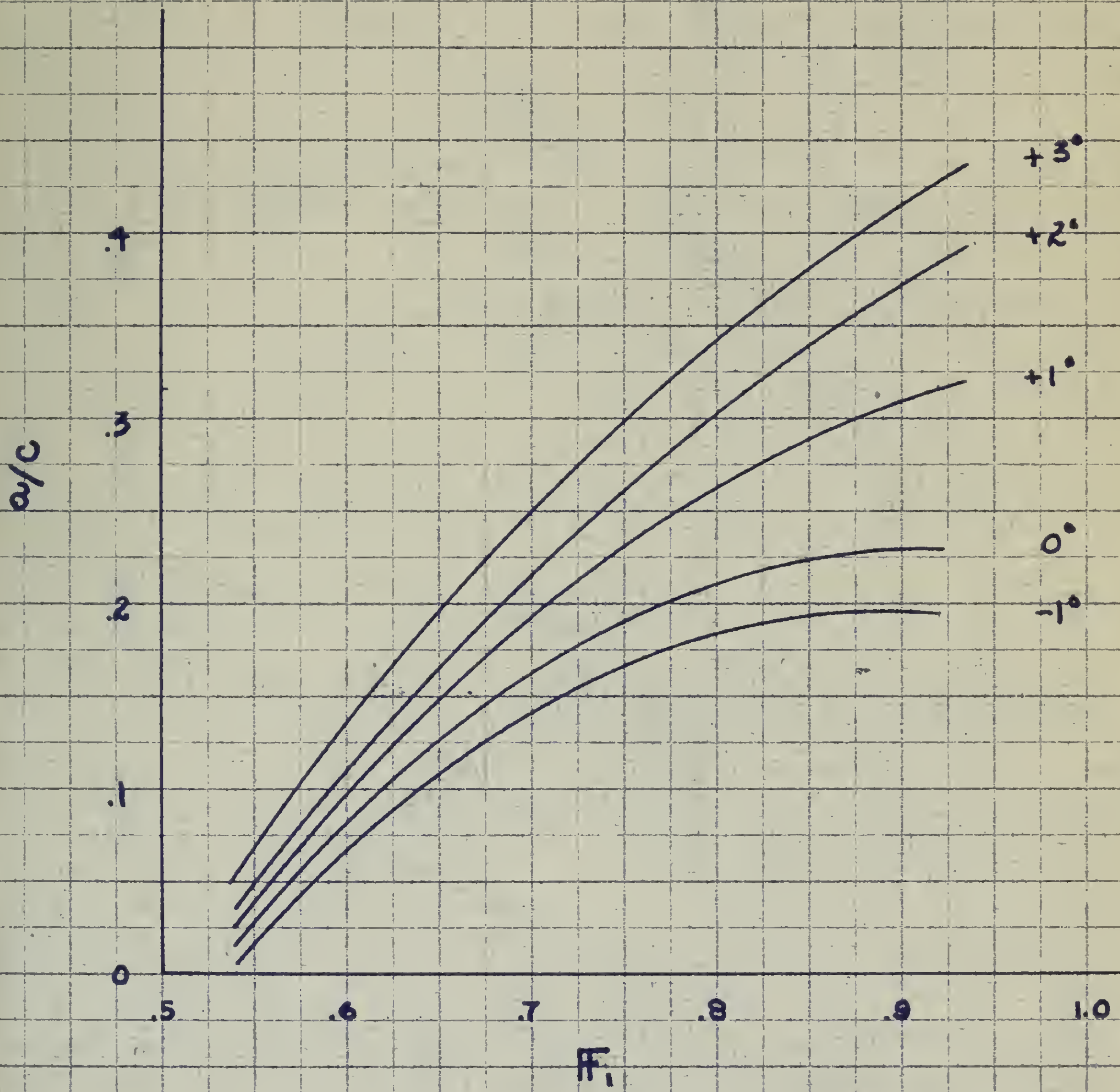
FIGURE VII


PLOT OF a/c vs. F_1
AT A DEPTH OF SUBMERGENCE
OF 1.2 C
AT VARIOUS ANGLES OF ATTACK
C.E. JONES, JR.
W.H. BROOKS, JR.
+3°
+2°
+1°
0°
-1°
.5
.4
.3
.2
.1
0
a/c
.5
.6
.7
.8
.9
1.0
F_1
25 MAY 1953
CEJ WHB

PLOT OF a/c VS. $\mathbb{F}_1$
AT A DEPTH OF SUBMERGENCE
OF 1.4 C
AT VARIOUS ANGLES OF ATTACK

C. E. JONES, JR.
W. H. BROOKS, JR.



25 MAY 1953

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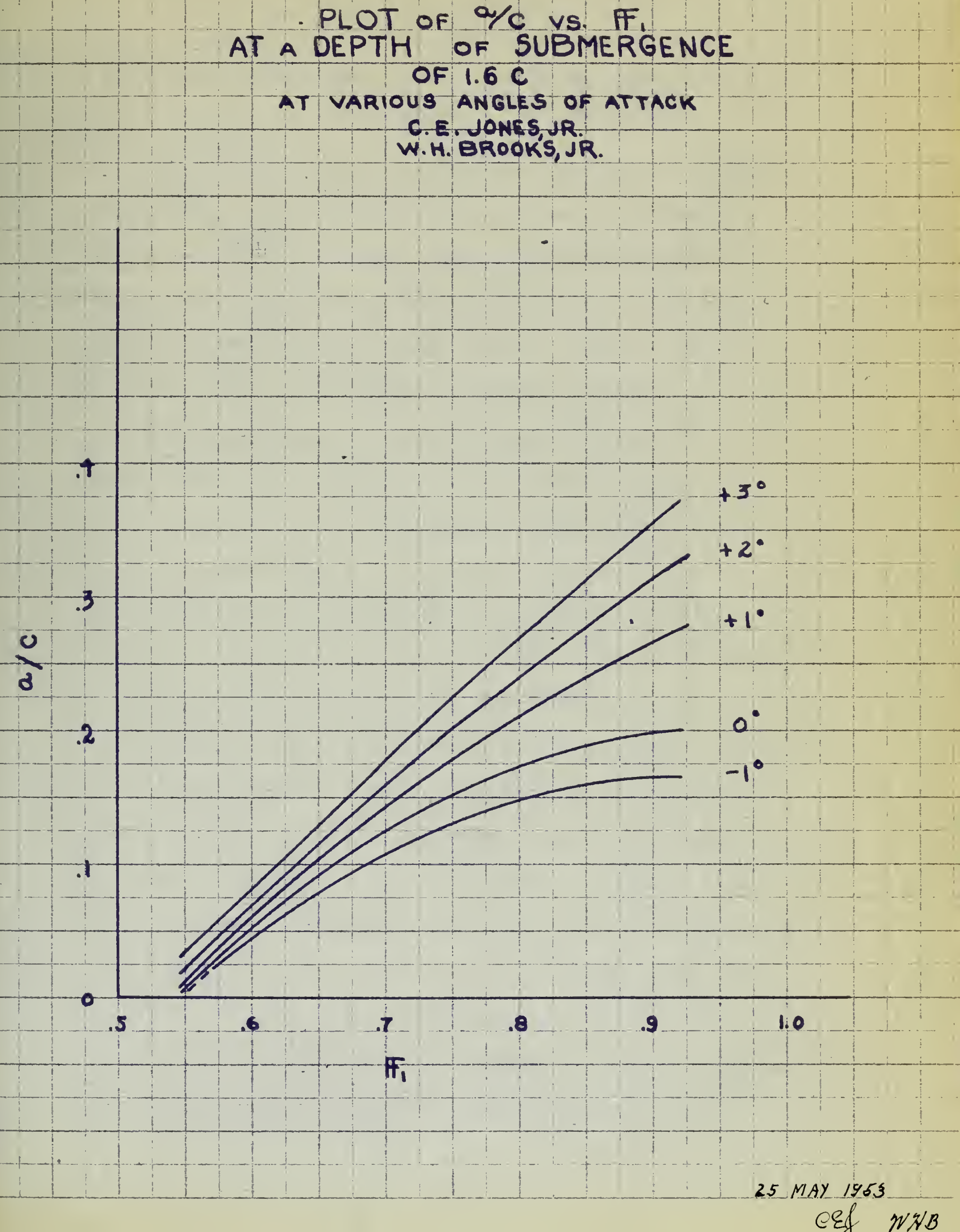
PLOT OF a/c VS. F₁
AT A DEPTH OF SUBMERGENCE
OF 1.6 C
AT VARIOUS ANGLES OF ATTACK
C. E. JONES, JR.
W. H. BROOKS, JR.
+3°
+2°
+1°
0°
-1°
.4
.3
.2
.1
0
a/c
.5
.6
.7
.8
.9
1.0
F₁
25 MAY 1953
CEJ WHB

IV DISCUSSION OF RESULTS

The object of this research was to find the change in the resulting wave pattern with changes in the conditions of the wave generating hydrofoil. In many cases, the relations were clearly defined and these have been expressed in the preceding section. In other instances (the parameters of the transition zone in particular), where such variations were not clearly defined, interpretation was needed. In these cases, we have made our interpretation and drawn our conclusions. For those who do not agree, the table, Summary of Data, found in appendix B, must comprise the results. It is hoped that sufficient information is expressed therein to aid the research of those individuals interested in hydrofoils.

Since this work was attempted with a view toward the use of hydrofoils in suppression of the wave train of a surface ship, it will be evaluated and interpreted primarily on this basis.

Not previously mentioned in this paper are the serious limitations imposed by the equipment which was used. Principal among them was an overall limitation on range of both velocity and total depth. At the present time, hydrofoils used as wave suppressors have been of chord-length equal to or greater than the draft of the ship model tested. This would mean, in full scale terms, that a destroyer type vessel, operating at a speed of about thirty knots, and using a foil with chord-length of about fifteen feet would give a Froude number on the foil ($V/\sqrt{gc}$) of about 2.3.

For a typical merchant vessel with a speed of about twenty knots, using a chord-length of twenty feet, the hydrofoil's Froude number might be 1.3. In our test range, the maximum Froude number attained for the foil was 0.92.

This means that the range of experimentation may not allow direct scaling of the results of these tests to a full size ship. This limit was recognized early in the preliminary analysis, and the use of three sizes of foils of about one, two, and three inches was planned. Manufacturing difficulties prevented the use of more than one foil. It is to be noted that the original proposal would have extended the range of Froude number to a value of 1.7. The extension of the range of Froude numbers tested is recommended in order that the scaling methods of ship model testing may be directly applied.

The use of a range of sizes of foils would also have permitted examination of one more variable, chord-length, and would have allowed a check on the validity of the use of Froude number for a body which does not penetrate the surface. Only limited work has been done on this aspect, but it appears that this is the valid scale factor within the limiting assumptions accepted in ship model testing.

Unfortunately, this work makes no contribution to the problem of determining the shape of surface waves. Attempted analysis of wave shape by numerical methods[3] proved unsuccessful. Thus, within the frame of characteristic dimensions presented here, the investigator or ship designer must choose which of the many pro-

posed theories he desires to use to express the shape of the wave.

No exact correlation of the characteristic dimensions of the transition was found. Examination of these dimensions indicates that their primary relation is to the respective dimensions of the steady state wave. In other words, $\ell 1$ and $\ell 2$ are increased by any change which increases λ, and y_0 is increased similarly with increases in "a". In the absence of any better correlation, numerical averages and the range of values obtained are presented. It should be noted that, for waves in which there was breaking of the crest of the first wave, the measured values of neither $\ell 2$ or of "a" were usable in this analysis. This is easily explained; the breaker is a region of high energy dissipation which affects the energy content of the wave which follows the breaker. Also of interest may be the fact that the presence of a breaker tends to stabilize the wave which follows, a d very good measurements of λ are possible.

During the observation of the hydro-dynamic behavior of the hydrofoil two important phenomena are noteworthy of mention. First, it was noted that within the velocity range of the investigation there was no perceptible influence on the upstream flow in front of the leading edge of the hydrofoil at distances greater than one chord-length. Second, it was noted that at shallow depths of submergence (d_1) a wave hump appeared above the leading edge of the hydrofoil. These two

phenomena are of importance to the ship designer if hydrofoils are to be used to reduce the wave making of a ship. The usefulness of the first phenomena is obvious in that the designer can rely on a free uninterrupted flow up a distance of one chord-length in front of the hydrofoil. The second phenomena, characterized by the dimension y_1, is not clearly defined at the present time. Only one conclusion is drawn--namely that in no case does a hump appear until the depth of submergency (d_1) decreases below .95c. This fact is useful to the designer in that within the Froude number range of this investigation it indicates that the designer should not locate a hydrofoil at the bow of a ship closer than one chord-length to the surface. Otherwise the presence of the hump will tend to offset the wave reducing feature of the hydrofoil.

The results of this investigation show that not only can a flume be used to study the behavior of a hydrofoil in deep water but also that the shallow water effect can easily be discerned by noting variation in wave-lengths with stream depth at various stream velocities. The resulting curves showing this phenomena need no further amplification.

Fortunately, theory and previous experimental work are available with respect to wave length of a surface wave. That the accepted criteria of λ for waves travelling over the surface of a deep body of water was met in the experimental results is a verification of the proposition that deep water conditions can be simulated in a relatively shallow, circulating water channel.

Since the generated waves are prone to be more unstable in horizontal location than in amplitude, an appreciable spread of experimental points about the mean line resulted.

With reference to wave stability, a/λ greater than 0.085 could not be achieved because of the breaking of the first wave crest. Following this stabilizing breaker a/λ values as high as 0.092 were achieved.

The relation of amplitude to submergence and angle of attack of the hydrofoil represents the primary result of this investigation. In the absence of comparative theory, the curves picturing this variation must be taken at the value indicated by the small deviations from the mean curves. These are design curves, and in addition lead to the following conclusion: for any angle of attack, there is an optimum depth which will produce the greatest amplitude of generated wave. This depth, in itself, if a function of the angle of attack, increasing with increased angle of attack in the manner shown by the curves of a/c vs. d_1/c at the various angles of attack.

This same information, plotted as a/c vs F_1, is presented, since this form may well be more useful in design.

V CONCLUSIONS

1. This range of experimentation may not allow the results of these tests to be scaled directly to a full sized ship.

2. No clearly defined relationships for the transition region were found.

3. Bow hydrofoils should not be installed at depths of submergence of less than about one chord-length.

VI RECOMMENDATIONS

1. Extension of the range of F_1 by similar tests to a value of approximately 2.5.

2. The use of geometrically similar foils of a different size to examine the variable, chord-length, and to check the validity of Froude Number as a scale factor.

Experimental Recommendations:

From observations and difficulties experienced, these recommendations are made for refinement of experimental techniques.

1. Insert false side walls in the flume to extend the velocity range and the depth of the flow. This would permit a better attachment of the hydrofoil to the sides and thus eliminate any "strut effect" which is set up by the side supports of the foil.

2. Install a fully adjustable weir gate to permit more precise control of velocity.

3. A more precise method for establishing angle of attack should be used. The present method, with the equipment used in this investigation, requires that the hydrofoil assembly be removed each time and set for a different angle of attack when so desired.

VII APPENDIX

APPENDIX A

DETAIL PROCEDURE AND DESCRIPTION OF EQUIPMENT

[illegible]

[illegible]

[illegible]

DETAILED PROCEDURE

Preliminary Analysis:

The closest approach to a theory for this problem is the work of Lamb[4] on surface waves due to a moving pressure disturbance. Though this theory was advanced considering a pressure disturbance at the surface, it was felt that the general method of attack is applicable to disturbances caused by a submerged body. In brief, the theory predicted (a) change of surface elevation over the finite length of the disturbance with the shape of the elevation closely linked to the nature of the disturbance, (b) a transition region of approximately ½ a wave length whose nature was exponential, (c) a damped oscillatory wave. Further, Lord Kelvin had theorized and experimentally checked the fact that no stable wave pattern would result at velocities of less than about 23 cm/sec.[5]

With this background, the method of procedure as stated on page 4 was adopted.

Also apparent in the preliminary analysis was the question of simulating deep water conditions in a channel of this type.

Only this far could theory help; it was necessary to know the general nature of the waves before proceeding.

Sequence of Investigation:

In order to get consistent results, reasonably steady approach flow to the foil had to be achieved. It was considered that this could be accomplished without modification to the channel, as installed, by placing our foil and observation section a distance of about fourteen feet from the inlet. Upon establishing flow with the sluice gate removed from the inlet, it was found that a standing wave existed the entire length of the channel. This difficulty was resolved by lowering the

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sluice gate to create a small head (one to two inches above the level of surface flow) in the inlet tank. This slight contraction of the inlet removed the standing wave, and the length of approach flow was sufficient to take care of the additional surface disturbance caused by the sluice gate.

With satisfactory approach flow established, the foil was placed in the flow; and velocity, submergence and angle of attack were varied to note qualitative effects. Two results were immediately noted: 1) The damping of the wave train was not discernable to the eye; 2) Lord Kelvin's prediction as to minimum velocity was optimistic in terms of stability of this equipment. Only random disturbance was present below a velocity of 1.25 feet per second, and, up to velocities of about 1.4 feet per second, measurement would be difficult.

It was also observed that the foil supports were creating some surface disturbance, and that this disturbance converged at the center of the channel at approximately the second wave crest, independent of the velocity of flow. To the eye, these effects appeared sizable, so it was decided to round the corners of the side supports, and if this did not suffice, to measure the surface profile generated by the supports alone, and try to subtract these values out of the profiles generated by the hydrofoil.

The rounding of the edges of the support pieces produced no change in the size of these disturbances, but, when the supports alone were placed in the flow, there was little or no effect on the surface, and no measurable wave was caused by these supports. Thus, it was decided that the unwanted surface disturbance was being caused, not by the side supports themselves, but by the complex intersection of foil,

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The rounding of the edges of the support plates produced no change in the size of these disturbances, but, when the supports alone were placed in the flow, there was little or no effect on the surface, and no measurable wave was caused by these supports. Thus, it was decided that the unwanted surface disturbance was being caused, not by the side supports themselves, but by the complex interaction of foil,

wall, and support. This was confirmed by observation that side effect first appeared at the surface at a different horizontal location as depth of the foil was changed (moving downstream, as submergence was increased).

As this could not be eliminated without major changes in the mounting, and it was felt that foil and channel wall alone would produce considerable effect even if the supports were removed from the flow, this three dimensional effect remained throughout the experimentation. However, as plotting of the profiles progressed, it was shown that this effect, though tending to make measurement difficult in the vicinity of the second crest, could be averaged out by careful use of the depth gage. Even with very closely spaced profile points, no appearance of this disturbance could be noted on the plotted profiles at the point where visual observation showed that side effects were present in the centerline profile.

A much more harmful effect of this side effect was its contribution to wave instability, particularly at low velocities, and, if a mounting could be designed to eliminate or reduce these disturbances, a minimum velocity much closer to that predicted by Lord Kelvin might be achieved.

Since this investigation would have been of little value unless it could be related to hydrofoil performance in deep water, it was felt that the first objective must be to determine the effects of the comparatively shallow channel. This was done as follows:

At each of several selected velocities, the controllable variables (angle of attack, depth of submergence, and velocity) were held constant while total depth was varied from the maximum allowed by pump

capacity to the minimum where total depth was only slightly greater than the depth of submergence. Resulting changes in the characteristics of the generated wave were then the "shallow water effects".

Once the point at which changes occurred was established, investigation proceeded at depths greater than this critical depth and thus deep water runs were simulated.

In addition, the first part of the investigation yielded enough data free of shallow water effects that it was possible to establish the fact that wave length was a function of velocity only.

The remainder of the investigation comprised the collection of sufficient data to establish the effects of α, d_1, and V on the transition zone and on amplitude of the steady wave.

DESCRIPTION OF EQUIPMENT

THE HYDROFOIL

The hydrofoil selected for this experiment was the N.A.C.A. 4412 airfoil section. The profile of this section is shown in the appendix Table no. I with a tabulation of its coordinates. The choice of this particular profile was based primarily on the availability of existing data of a similar nature which would be useful in the course of this investigation.[1][2] This airfoil is 2.8"in chord length and 18" in wing length. It has a 12 percent thickness ratio with a 4 percent camber. The trailing edge was rounded off slightly to facilitate the machining of the foil. The foil was made of dural and was manufactured by a special milling machine in the Sloan Laboratory of the Institute.

THE WATER CHANNEL

A photograph of the water channel is shown in Figure X . This flume is capable of a maximum flow rate of 1200 G.P.M. It is 18" in width and 24 feet in length. The entrance of the channel contained radiator baffling followed by a converging section to stabilize the channel flow rate and produce as little surface disturbance as possible. For the low velocities needed for this investigation (about 1-2 F.P.S.) this arrangement was not quite satisfactory because standing waves were generated on the upstream side of the test section. A satisfactory flow surface was produced by merely lowering the sluice gate until it made contact with the water surface. Surface disturbance variation, (unsteady) was,

at the most, about $1\frac{1}{2}$ millimeters.

The flow rate of the channel was established by means of a calibrated orifice section located in the piping on the discharge side of the pump. A differential mercury manometer was attached to this section and the flow rate could be obtained by measuring the difference in mercury levels. Then applying the calibration formula

$$Q = 1.806 \sqrt{H} \tag{2}$$

the flow rate and hence the velocity could be determined.

THE MEASURING EQUIPMENT

A depth probe, calibrated in centimeters, was used to measure the heights of the generated waves, the undisturbed stream surface and to establish the vertical height of the foil tip. The probe was mounted on a carriage which could be moved on rails located on top of the flume.

A telescope apparatus shown in Fig. XI was used in measuring the horizontal distances. Due to the long arm on the depth probe, inaccurate horizontal distance readings would result if horizontal distances were measured using the probe carriage. The telescope was mounted on a moving slider which was free to move in a horizontal direction along an aluminum 2 x 2 x $\frac{1}{4}$ " angle. This aluminum angle was securely bolted at each extremity of the observation area and checked for level and cross-level by means of a machinist's level.

A steel tape was laid along the top surface of the angle for use in measuring horizontal distances. Vertical movement of the telescope was achieved by clamping the telescope on a depth probe which in turn was mounted on the slider.

The procedure in using the telescope apparatus is as follows: the slider is first moved to a desired position on the angle bar. Then the probe carriage is moved along the length of the flume until the probe tip is observed in the cross hair of the telescope. This establishes the abscissa of the particular point on the wave profile to be measured.

TABLE I

TABLE OF OFFSETS FOR NACA 4412

AIRFOIL SECTION

STATION	UPPER	LOWER
0	-	0
1.25	2.44	-1.43
2.5	3.39	-1.95
5.0	4.73	-2.49
7.5	5.76	-2.74
10.0	6.59	-2.86
15.0	7.89	-2.88
20.0	8.80	-2.74
25.0	9.41	-2.50
30.0	9.76	-2.26
40.0	9.80	-1.80
50.0	9.19	-1.40
60.0	8.14	-1.00
70.0	6.69	-0.65
80.0	4.89	-0.39
90.0	2.71	-0.22
95.0	1.47	-0.16
100.0	0.13	-0.13
100.0	-	0

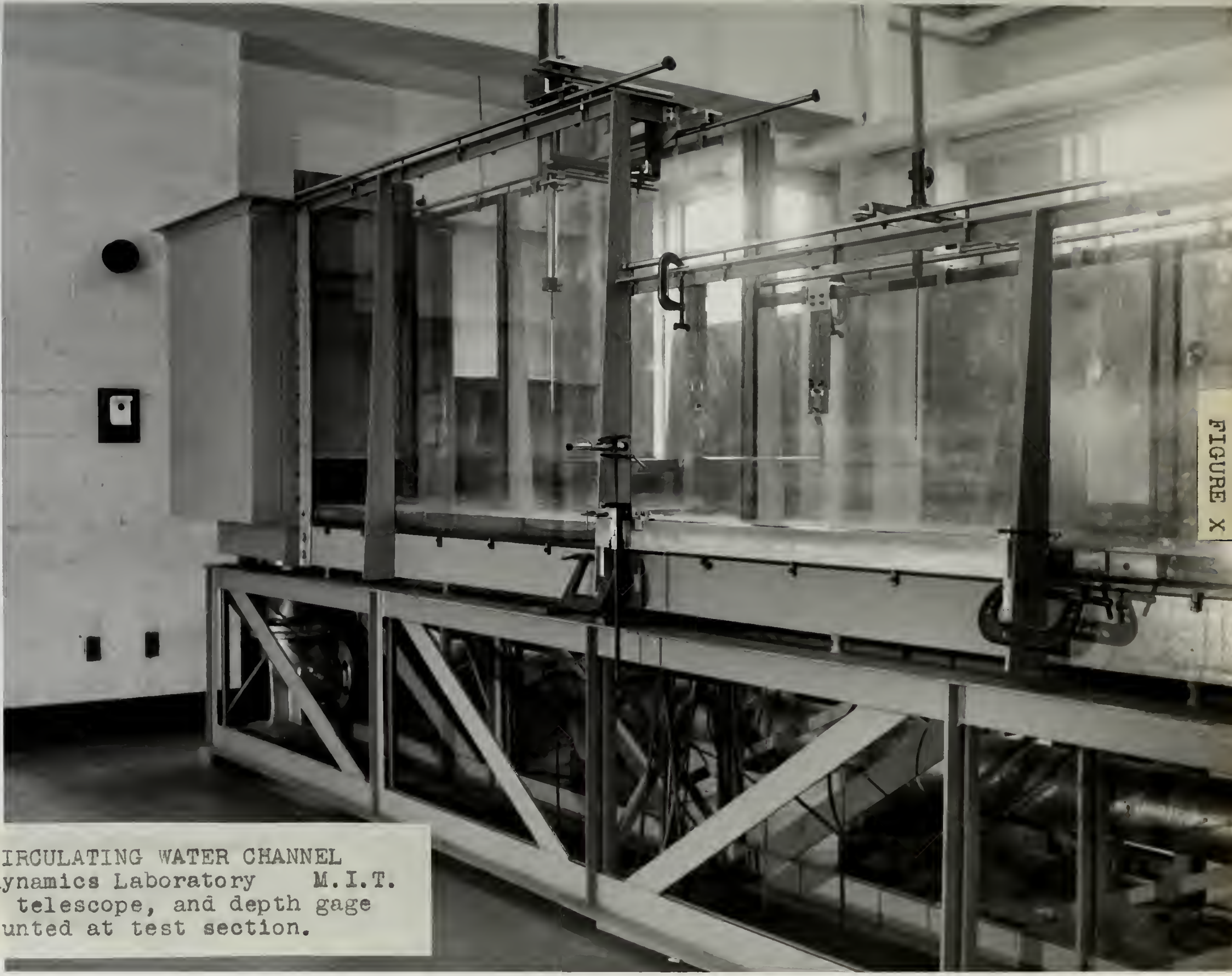
Leading edge radius = 1.58
Slope of radius through end of chord = 4/20
Max. mean camber = .04 x C
Location of max. mean camber = .4 x C

Max. thickness = .12 x C

In the above table the stations are expressed as percentages of the chord length. The ordinates to the upper and lower surfaces are also expressed as percentages of chord length.

The foil used in this investigation was designed for a chord length of 2.80 inches. The trailing edge had to be rounded off slightly to facilitate machining. The actual measured chord length was 2.77 inches.

FIGURE X



CIRCULATING WATER CHANNEL
Hydrodynamics Laboratory M.I.T.
Foil, telescope, and depth gage
mounted at test section.

FIGURE XI

TELESCOPE
Close-up showing
mount and bench.

FIGURE XII

HYDROFOIL
Close-up of foil
and movable mount.

FIGURE XIII

FOIL IN OPERATION

$\alpha = 0^{\circ}$ $\lambda = 9.4$ in.

$V = 2.0$ ft/sec $a = 0.55$ in.

$d_1 = 3.46$ in.

$d_o = 8.27$ in.

FIGURE XIV

FOIL IN OPERATION

α = 0° λ = 14.6 in.
V = 2.5 ft/sec a = 0.78 in.
d_1 = 2.08 in.
d_o = 7.68 in.
Note surface rise above hydrofoil.

APPENDIX B

SUMMARY OF DATA AND CALCULATIONS

TABLE II

SUMMARY OF EXPERIMENTAL DATA

RUN NO.	d_1 in.	d_o in.	α o	V ft/sec	λ in.	a in.	y_o in.	y_1 in.	l_1 in.	l_2 in.
7	2.94	9.95	2	1.30	4.5	0.17	0.13	-	2.2	5.0
8	2.92	10.24	2	1.83	8.0	0.50	0.37	-	3.9	R
9	2.94	6.72	2	1.40	5.7	0.42	0.23	-	2.6	5.6
10	2.93	9.03	2	1.72	7.3	0.61	0.30	-	3.8	7.6
11	2.97	7.77	2	1.60	6.1	0.44	0.24	-	2.9	6.6
12	2.94	8.70	2	1.19	4.1	0.13	0.16	-	2.2	4.6
13	2.94	7.20	2	1.98	8.4	0.72	0.43	-	4.2	R
14	2.90	4.75	2	1.32	5.5	0.34	0.16	-	2.3	5.7
15	2.99	6.44	2	1.29	4.8	0.29	0.18	-	2.3	5.5
16	2.93	7.70	2	1.31	5.0	0.24	0.16	-	2.7	5.4
17	2.93	6.72	2	2.51	16.0	1.30	0.64	-	6.5	14.2
18	2.91	4.44	2	2.51	19.3	1.46	0.77	-	7.6	17.0
19	2.97	5.57	2	2.48	16.0	1.40	0.66	-	6.0	14.7
20	2.54	6.95	2	2.24	12.0	1.10	0.45	-	5.2	R
21	3.01	6.32	2	2.03	9.2	0.61	0.43	-	4.0	R
22	3.02	8.4	2	1.99	9.2	0.62	0.47	-	3.9	R
23	2.94	9.2	2	1.97	8.6	0.58	0.37	-	4.2	R
24	2.93	4.66	2	2.01	11.0	0.37	0.52	-	5.2	R
25	2.03	3.53	2	2.00	12.2	0.85	0.63	-	5.3	R
26	2.00	7.5	2	1.84	8.3	0.48	0.41	-	4.4	R
27.5	2.00	7.5	2	2.06	10.1	0.85	0.47	-	5.0	R
28	2.00	7.5	2	2.30	12.0	1.02	0.47	-	6.0	R
29a	1.00	8.31	0	2.00	9.2	0.70	0.26	0.28	4.6	9.5
29b	1.76	8.31	0	2.00	9.4	0.76	0.27	0.16	4.6	9.5
29c	2.50	8.31	0	2.00	9.4	0.73	0.30	-	4.6	9.5
30a	3.51	8.31	0	2.00	10.0	0.57	0.21	-	4.6	9.5
30b	4.50	8.31	0	2.00	10.0	0.40	0.17	-	4.6	9.5
31a	4.88	8.31	2	2.00		0.42	0.22	-	4.2	9.7
31b	4.13	8.31	2	2.00		0.60	0.32	-	4.2	9.7
31c	3.44	8.31	2	2.00		0.78	0.38	-	4.2	9.7
36a	3.85	8.31	3	2.00		0.79	0.45	-	4.2	9.6
36b	4.98	8.31	3	2.00		0.42	0.28	-	4.2	9.6
36c	4.36	8.31	3	2.00		0.57	0.34	-	4.2	9.6
37a	2.64	8.31	1	2.00		0.81	0.43	-	4.2	9.7
37b	3.15	8.31	1	2.00		0.75	0.45	-	4.2	9.7
37c	3.86	8.31	1	2.00		0.62	0.36	-	4.2	9.7
37d	4.70	8.31	1	2.00		0.43	0.21	-	4.2	9.7
38a	4.96	8.31	-1	2.00		0.26	0.16	-	4.4	9.7
38b	4.05	8.31	-1	2.00		0.41	0.20	-	4.4	9.7
38c	3.15	8.31	-1	2.00		0.58	0.28	-	4.4	9.7
38d	2.25	8.31	-1	2.00		0.67	0.32	0.06	4.4	9.7
39	1.16	8.31	-1	2.00		0.64	0.26	0.22	4.4	9.7
40a	4.75	7.65	3	2.51		0.91	0.43	-	6.3	15.4
40b	3.77	7.65	3	2.51		1.20	0.55	-	6.3	15.4

TABLE II
(cont'd)

RUN NO.	d_1 in.	d_o in.	α o	V ft/scs	λ in.	a in.	y_o in.	y_1 in.	l_1 in.	l_2 in.
40c	2.76	7.65	3	2.51		1.34	0.63	-	6.3	15.4
40d	1.80	7.65	3	2.51		1.24	0.71	0.03	6.3	15.4
41a	0.82	7.65	3	2.51		0.92	0.49	0.24	6.3	15.4
41b	4.78	7.65	2	2.51		0.80	0.45	-	6.3	15.2
41c	3.77	7.65	2	2.51		1.09	0.52	-	6.3	15.2
41d	2.80	7.65	2	2.51		1.26	0.63	-	6.3	15.2
41e	1.80	7.65	2	2.51		1.18	0.57	0.08	6.3	15.2
41f	0.81	7.65	2	2.51		0.77	0.43	0.19	6.3	15.2
41g	4.80	7.65	1	2.51		0.70	0.39	-	6.4	15.3
41h	3.81	7.65	1	2.51		0.91	0.45	-	6.4	15.3
42a	2.84	7.65	1	2.51		1.05	0.49	0.04	6.4	15.3
42b	1.82	7.65	1	2.51		1.05	0.52	0.12	6.4	15.3
42c	0.84	7.65	1	2.51		0.74	0.38	0.20	6.4	15.3
42d	4.85	7.65	0			0.50	0.33	-	6.3	15.2
42e	3.85	7.65	0	2.51		0.65	0.37	-	6.3	15.2
42f	2.85	7.65	0	2.51		0.74	0.43	0.06	6.3	15.2
42g	1.85	7.65	0	2.51		0.78	0.45	0.10	6.3	15.2
42h	0.86	7.65	0	2.51		0.63	0.28	0.18	6.3	15.2
43a	4.85	7.65	-1	2.51		0.41	0.30	-	6.4	15.6
43b	3.86	7.65	-1	2.51		0.54	0.35	-	6.4	15.6
43c	2.84	7.65	-1	2.51		0.67	0.37	0.04	6.4	15.6
43d	1.85	7.65	-1	2.51		0.71	0.37	0.14	6.4	15.6
43e	0.87	7.65	-1	2.51		0.52	0.26	0.24	6.4	15.6
44a	2.98	9.65	3	1.50						
44b	3.72	9.65	3	1.50						
44c	4.50	9.65	3	1.50						
44d	3.42	9.65	3	1.50						
44e	3.01	9.65	2	1.50						
44f	3.51	9.65	2	1.50						
44g	4.04	9.65	2	1.50						
44h	3.02	9.65	1	1.50						
44i	3.54	9.65	1	1.50						
44j	4.05	9.65	1	1.50						
44k	2.57	9.65	0	1.50						
44l	3.04	9.65	0	1.50						
44m	3.54	9.65	0	1.50						
44n	2.55	9.65	-1	1.50						
44o	3.04	9.65	-1	1.50						
44p	3.58	9.65	-1	1.50						

This data not taken in following runs

NOTE: 1) R in l_2 column indicates breaking at first wave crest.
2) - in y_1 column indicates no surface rise directly above foil.

TABLE II
(cont'd)

Limits of accuracy in terms of probable error.

d_1	0.02	in.
d_o	0.02	in.
α	0.1°	
V	0.01	ft/sec.
λ	0.3	in.
a	0.05	in.
y_o	0.02	in.
y_1	0.02	in.
l_1	0.2	in.
l_2	0.4	in.

TABLE III

TABULATION OF DATA FOR THE PLOT OF THE VARIATION OF WAVE-LENGTH WITH DEPTH OF WATER TO NOTE SHALLOW-WATER EFFECTS

$V = 1.30$ ft./sec. $\alpha = 2^\circ$ $d_1 = 2.940$ in.

Run	do(ft.)	V ft./sec.	V^2	λ ft.	λ (corrected)
7	0.829	1.300	1.69	0.375	0.375
9	0.560	1.485	2.21	0.475	0.363
10	0.752	1.720	2.96	0.608	0.347
11	0.647	1.595	2.55	0.508	0.337
14	0.396	1.320	1.75	0.458	0.413
15	0.536	1.294	1.68	0.396	0.398
16	0.641	1.310	1.72	0.416	0.408

$V = 2.00$ ft./sec. $\alpha = 2^\circ$ $d_1 = 2.940$ in.

Run	do	V	V^2	λ	λ (corrected)
13	0.600	1.975	3.90	0.700	0.718
20	0.579	2.240	5.02	1.000	0.798
21	0.526	2.030	4.13	0.766	0.742
22	0.700	1.990	3.96	0.766	0.774
23	0.718	1.972	3.89	0.716	0.736
24	0.388	2.010	4.04	0.917	0.908
25	0.294	2.000	4.00	1.017	1.017
30	0.689	1.980	3.93	0.750	0.764

$V = 2.5$ ft./sec. $\alpha = 2^\circ$ $d_1 = 2.94$ in.

Run	do	V	V^2	λ	λ (corrected)
17	0.559	2.510	6.32	1.333	1.319
18	0.370	2.510	6.32	1.608	1.59
19	0.464	2.480	6.15	1.333	1.356
40	0.636	2.510	6.32	1.250	1.245

Velocities of 1.3, 2.0, and 2.5 ft./sec. were chosen to show on the curve because most of the runs made during the early part of the investigation were at velocities near these. As shown above all of the wave-lengths were corrected by proportioning based on the ratio of the square of the velocities

$$\frac{\lambda_1}{\lambda_2} = \left(\frac{V_1}{V_2}\right)^2 \qquad (3)$$

which was previously established.

TABLE IV

TABULATION OF DATA FOR THE PLOT OF VELOCITY VS WAVE-LENGTH FOR DEEP WATER OPERATION

RUN	V	λ''	λ'
8	1.83	8.00	.666
10	1.72	7.30	.608
11	1.595	6.10	.508
17	2.51	16.00	1.334
20	2.24	12.00	1.00
22	1.99	9.20	.766
23	1.97	8.60	.716
26	1.835	8.30	.692
26.5	2.45	14.0	1.165
29-30	1.98	9.6	.800
40	2.48	15.00	1.25

The following calculations serve to substantiate the theory. From the curve of velocity versus wave-length the following points were taken:

$\lambda = 1.76$ ft. $\qquad V = 3.00$ ft./sec.
$\lambda = 0.44$ ft. $\qquad V = 1.50$ ft./sec.

The equation is of the form:

$$\lambda = KV^n \tag{4}$$

$$(1.76 = K(3.00)^n$$
$$(0.44 = K(1.50)^n$$
$$\ln\lambda = \ln K + n(\ln V)$$

$$\ln 1.76 = \ln K + n \ln 3.00$$
$$\ln 0.44 = \ln K + n \ln 1.50$$

$$0.565 = \ln K + n\, 1.098$$
$$-0.823 = \ln K + n\, 0.405$$
$$1.388 = 0.693\, n$$
$$\therefore \quad n = 2$$

$$0.565 = \ln K + 2\,(1.098)$$
$$\ln K = -1.631$$
$$K = 0.195$$

$$\therefore \quad \lambda = 0.195\, V^2 = \frac{2\pi}{g} V^2$$

TABLE V

TABULATION OF DATA FOR PLOTS OF a/c VERSUS d_1/c
AT VARIOUS ANGLES OF ATTACK

$V =$ 2.51 ft./sec. $\quad F_1 = \frac{V}{\sqrt{gc}} =$ .920 $\quad c =$ 2.77 in.

α = -1°		α = 0°		α = 1°		α = 2°		α = 3°	
a/c	d_1/c	a/c	d_1/c	a/c	d_1/c	a/c	d_1/c	a/c	d_1/c
0.188	0.314	0.228	0.310	0.267	0.303	0.278	0.292	0.332	0.296
0.256	0.669	0.282	0.669	0.379	0.657	0.426	0.650	0.448	0.650
0.242	1.025	0.267	1.029	0.379	1.025	0.455	1.010	0.484	0.996
0.195	1.395	0.235	1.390	0.328	1.375	0.394	1.360	0.433	1.360
0.148	1.750	0.180	1.750	0.253	1.735	0.289	1.725	0.328	1.715

$V =$ 2.00 ft./sec. $\quad F_1 =$ 0.733 $\quad c =$ 2.77 in.

α = -1°		α = 0°		α = 1°		α = 2°		α = 3°	
a/c	d_1/c	a/c	d_1/c	a/c	d_1/c	a/c	d_1/c	a/c	d_1/c
0.231	0.419	0.253	0.361	0.292	0.954	0.311	1.060	0.285	1.390
0.242	0.813	0.274	0.635	0.271	1.138	0.282	1.242	0.206	1.575
0.210	1.139	0.264	0.903	0.224	1.393	0.217	1.490	0.152	1.798
0.148	1.462	0.206	1.267	0.155	1.697	0.152	1.760		
0.094	1.790	0.144	1.625						

$V =$ 1.50 ft./sec. $\quad F_1 =$ 0.550 $\quad c =$ 2.77 in.

α = -1°		α = 0°		α = 1°		α = 2°		α = 3°	
a/c	d_1/c	a/c	d_1/c	a/c	d_1/c	a/c	d_1/c	a/c	d_1/c
0.115	0.921	0.134	0.928	0.101	1.090	0.130	1.087	0.159	1.076
0.051	1.098	0.083	1.098	0.072	1.280	0.072	1.268	0.098	1.235
0.018	1.292	0.058	1.280	0.029	1.462	0.040	1.460	0.072	1.342
								0.032	1.625

TABLE VI

NUMERICAL AVERAGES RELATING THE

TRANSITION TO THE STEADY WAVE

Run	y_o/a(%)
31a	52
b	54
c	50
36a	57
b	67
c	59
37a	53
b	60
c.	58
d	49
38a	62
b	49
c	48
d	48
39	44
40a	47
b	46
c	47
d	57
41a	53
b	56
c	48
d	51
e	48
f	56
g	56
h	50
42a	47
b	50
c	51
d	66
e	57
f	58
g	58
h	44
43b	65
c	55
d	52
e	50
Avg.	53.3
Min.	44
Max.	67

Run	L_1/λ(%)
7	49
8	49
9	46
10	52
11	48
12	54
13	49
15	48
16	54
23	49
26	53
27.5	50
28	50
Avg.	50
Min.	46
Max.	54

Run	$\frac{L_2}{\lambda}$
7	1.11
9	0.98
10	1.04
11	1.08
12	1.11
13	1.04
15	1.14
16	1.08
29a	1.03
29b	1.01
29c	1.01
30a	0.95
30b	0.95
Avg.	1.04
Min.	0.95
Max.	1.14

TABLE VII

SAMPLE DATA SHEET (RUN 29a)

Manometer 1.30 feet, $\alpha = 0^{\circ}$
Foil Location: vertical 34.25 cm., horizontal 16.70 cm.

PROFILE DATA

VERTICAL	HORIZONTAL	VERTICAL	HORIZONTAL
36.75	11.00	35.88	31.00
36.76	13.00	35.97	32.00
36.79	14.00	36.56	33.00
36.92	15.00	37.41	34.00
36.98	16.00	37.75	35.00
37.42	17.00	37.19	36.50
37.47	17.40	36.14	38.00
37.61	18.20	35.86	39.50
37.01	19.00	36.08	41.00
36.57	20.00	36.87	42.50
36.25	21.00	37.36	44.00
36.10	22.00		
36.31	23.00		
36.69	24.00		
37.25	25.00		
37.75	26.00		
37.55	27.00		
37.09	28.00		
36.50	29.00		
36.15	30.00		

The above is data for one of the runs in which a complete profile was mapped. Vertical distances are recorded in centimeters and horizontal distances are recorded in inches. Bottom elevation was 15.75 cm.

TABLE VIII

SAMPLE DATA SHEET (RUN 41b)

Manometer 1.77 feet, $\alpha = 2^\circ$.
Foil location: vertical 23.09 cm., horizontal 16.40 cm.

VERTICAL (cm.)	HORIZONTAL (in.)
35.22	11.00
35.22	13.00
35.22	15.00
35.01	17.00
34.64	19.00
34.28	21.00
34.11	23.00
34.44	25.00
34.95	27.00
35.58	29.00
35.95	31.00
35.80	33.00
35.45	35.00

Max. (Vert. 33.92 cm.
(Hori. 40.40 in.

Min. (Vert. 35.82 cm.
(Hori. 48.80 in.

The above table is data for one of the runs of the series in which the transient was to be studied and the amplitude was to be noted.

TABLE IX

SAMPLE DATA SHEET (RUN 44a)

Manometer 1.015 feet, $\alpha = 3^{\circ}$, $d_o = 40.25$ cm.
Foil location: vertical 32.68 cm.

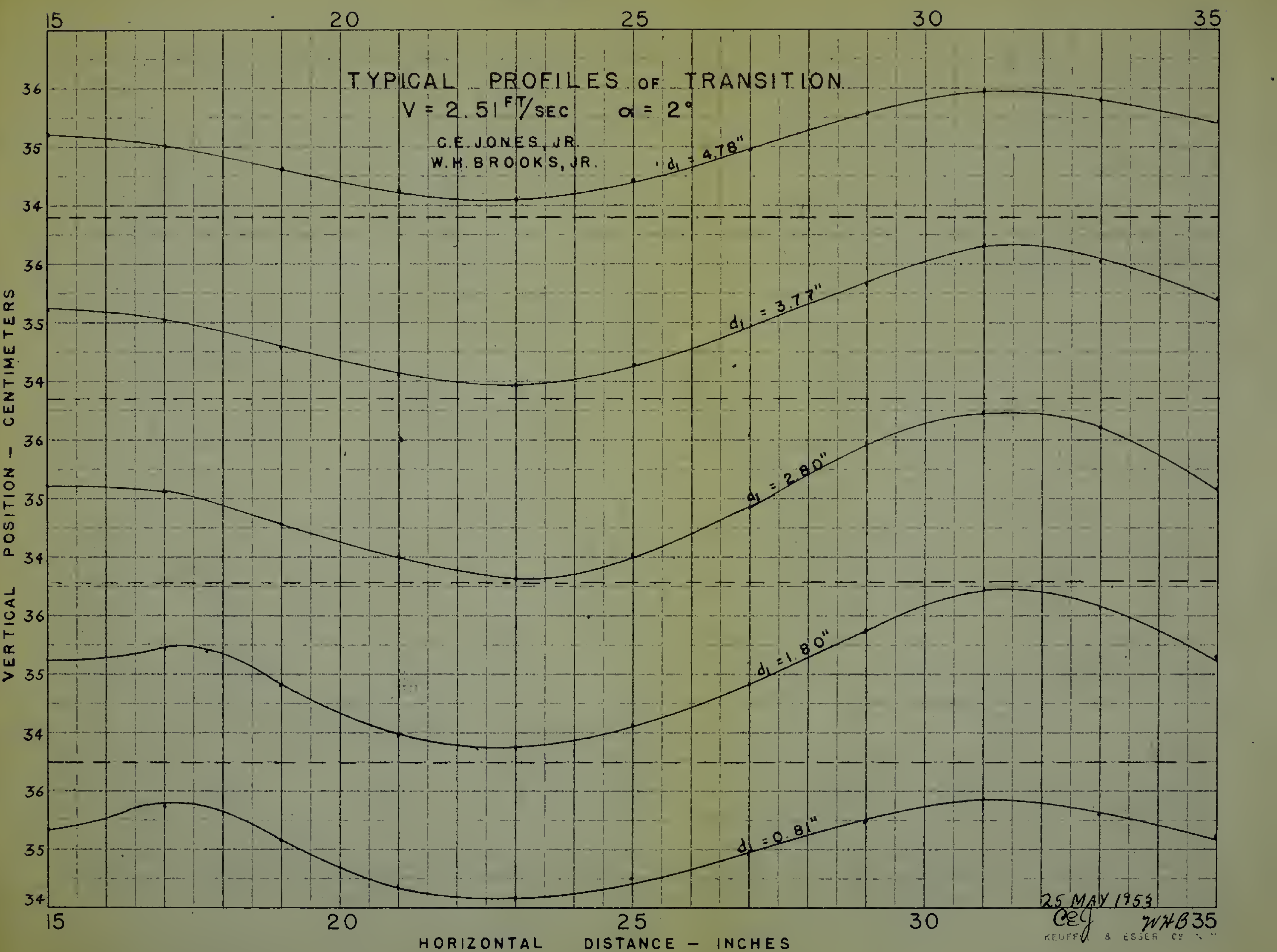
Max.	40.75	cm.
Min.	39.60	cm.
Max.	40.64	cm.
Min.	39.69	cm.
Max.	40.84	cm.
Min.	39.59	cm.

The above is data from a run of a series in which amplitude only was studied.

APPENDIX C

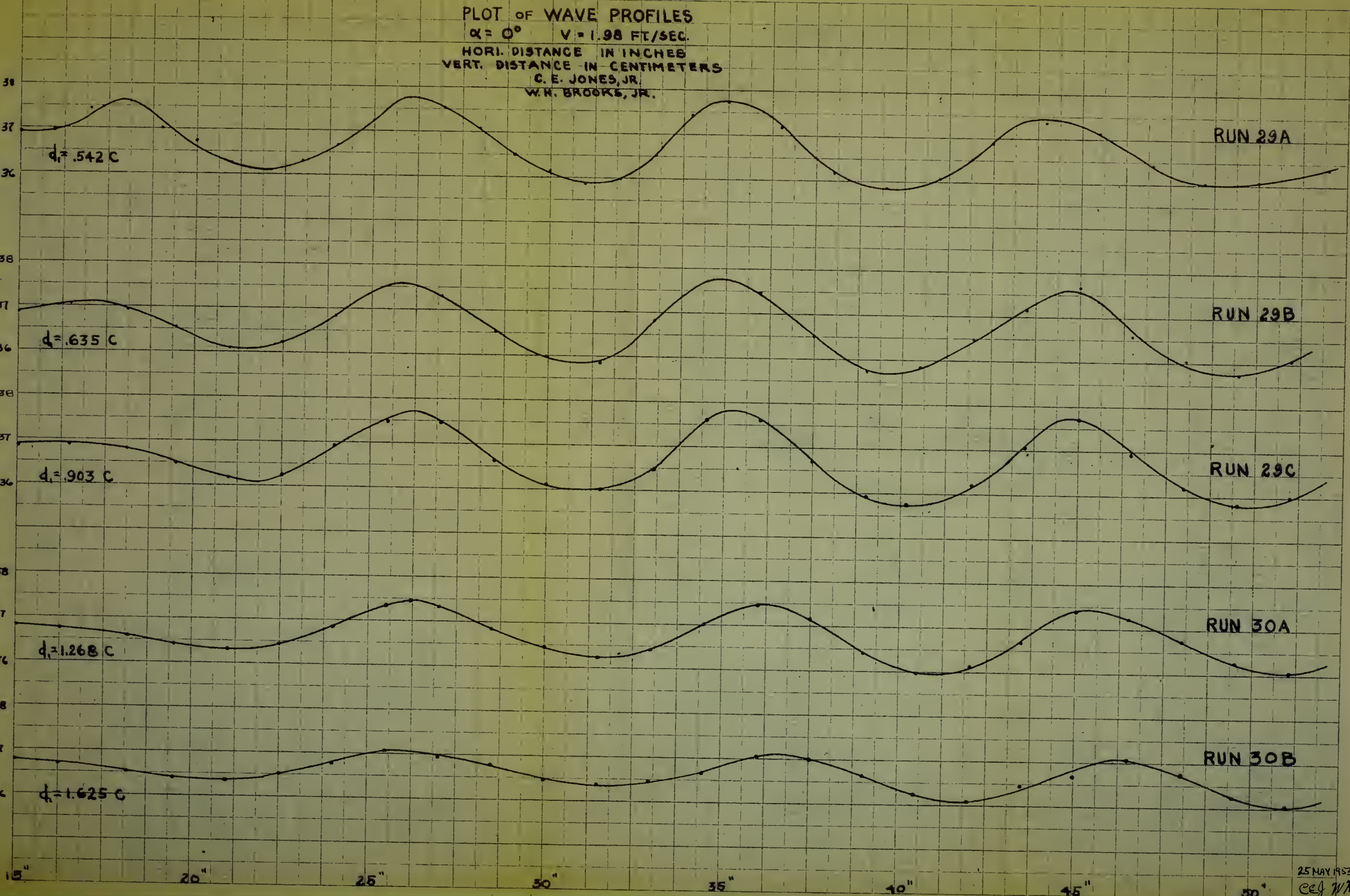
SAMPLE INTERMEDIATE PLOTS

FIGURE XV



TYPICAL PROFILES OF TRANSITION
V = 2.51 FT/SEC α = 2°
C.E. JONES, JR.
W.H. BROOKS, JR.
d1 = 4.78"
d1 = 3.77"
d1 = 2.80"
d1 = 1.80"
d1 = 0.81"
VERTICAL POSITION — CENTIMETERS
HORIZONTAL DISTANCE — INCHES
15
20
25
30
35
36
35
34
25 MAY 1953
CEJ
WHB
KEUFFEL & ESSER CO.

FIGURE XVI


PLOT OF WAVE PROFILES
$\alpha = 0°$ V = 1.98 FT/SEC.
HORI. DISTANCE IN INCHES
VERT. DISTANCE IN CENTIMETERS
C. E. JONES, JR.
W. H. BROOKS, JR.
RUN 29A
d_1 = .542 C
RUN 29B
d_1 = .635 C
RUN 29C
d_1 = .903 C
RUN 30A
d_1 = 1.268 C
RUN 30B
d_1 = 1.625 C
38
37
36
15"
20"
25"
30"
35"
40"
45"
50"
25 MAY 1953
CEJ WHB

APPENDIX D

LITERATURE CITATIONS

LITERATURE CITATIONS

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3. Scarborough, J. B., "Numerical Mathematical Analysis", Johns Hopkins Press. Pages 443-445.

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